Exercise 52

If $g(\theta) = \theta \sin\theta$, find $g''(\pi/6)$.

Solution

Calculate the first derivative of $g(\theta)$.

$$\begin{aligned} g'(\theta) &= \frac{d}{d\theta}(\theta \sin\theta) \\ &= \left[\frac{d}{d\theta}(\theta)\right]\sin\theta + \theta\left[\frac{d}{d\theta}(\sin\theta)\right] \\ &= (1)\sin\theta + \theta(\cos\theta) \\ &= \sin\theta + \theta\cos\theta \end{aligned}$$

Calculate the second derivative of $g(\theta)$.

$$\begin{aligned} g''(\theta) &= \frac{d}{d\theta}(\sin\theta + \theta\cos\theta) \\ &= \frac{d}{d\theta}(\sin\theta) + \frac{d}{d\theta}(\theta\cos\theta) \\ &= \cos\theta + \left[\frac{d}{d\theta}(\theta)\right]\cos\theta + \theta\left[\frac{d}{d\theta}(\cos\theta)\right] \\ &= \cos\theta + (1)\cos\theta + \theta(-\sin\theta) \\ &= 2\cos\theta - \theta\sin\theta \end{aligned}$$

Plug in $\theta = \pi/6$ to find $g''(\pi/6)$.

$$\begin{aligned} g''\left(\frac{\pi}{6}\right) &= 2\cos\left(\frac{\pi}{6}\right) - \left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right) \\ &= 2\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{6}\right)\left(\frac{1}{2}\right) \\ &= \sqrt{3} - \frac{\pi}{12} \\ &= \frac{12\sqrt{3} - \pi}{12} \\ &\approx 1.47025 \end{aligned}$$